guide section was only 3.5 db at 6.8 kMc. When the operating frequency is reduced further, the cutoff of Cohn's mode appears. In the case of Fig. 4, Cohn's cutoff is close to the actual cutoff of the channel waveguide.

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A Coaxial Adjustable Sliding Termination

INTRODUCTION

The accuracy of impedance measurements using modified reflectometer techniques depends mainly upon the tuning of the reflectometer. This tuning is accomplished by sliding first a low-reflection termination and, then, a high-reflection termination (sliding short circuit) in the output waveguide of the reflectometer. The actual error that can occur due to imperfect tuning can be computed[1] and depends to a large extent upon the size of the reflection coefficient of the low-reflection sliding termination. The lower this reflection coefficient is, the smaller the error will be. The adjustable sliding termination described in this paper was developed to reduce this reflectometer tuning error; hence, the main emphasis was on obtaining a stable, very low-reflection coefficient.

DESCRIPTION OF INSTRUMENT

A drawing showing the principle of the instrument is shown in Fig. 1. The principle of operation is similar to the one described by Ellenwood and Ryan.[2] The main difference is that, instead of using a double slug tuner in front of a terminating element, the reflection coefficient of the actual terminating element is variable. It is varied by moving a lossy cylinder inside of a lossy taper. When the face of the cylinder is positioned immediately in front of the edge of the taper, a maximum reflection is obtained; when it is completely withdrawn inside of the taper, a minimum reflection is obtained.

The total reflection coefficient of the sliding termination is a combination of the reflection from the terminating elements and the reflection from the bead in front of the terminating element. The bead is mounted on a very thin dielectric tube that extends

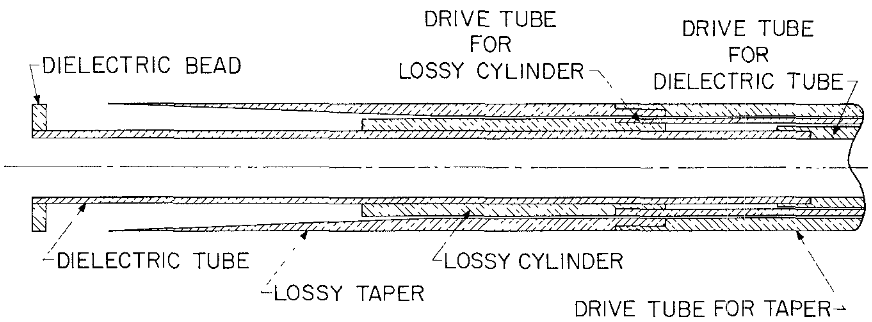

Fig. 1—Cross section of the adjustable sliding termination.

through the lossy terminating elements. A maximum reflection occurs when the two reflections are in phase, and a minimum reflection occurs when they are out of phase.

THEORY OF OPERATION

The equation[3] for the total reflection coefficient of the termination in terms of the reflection coefficients of the bead and the terminating elements is

$$\Gamma = \Gamma_1 + \frac{\Gamma_L(1 - \Gamma_1^2)e^{j\phi}}{1 + \Gamma_1\Gamma_L e^{j\phi}}, \qquad (1)$$

where Γ_1 is the reflection coefficient of the bead, Γ_L is the reflection coefficient of the terminating element and ϕ is the angle of combination of the two reflection coefficients. The assumption that the bead and transmission line are lossless has been made in order to use this equation.

The minimum value of Γ is

$$\Gamma_{\min} = \frac{|\Gamma_1| - |\Gamma_L|}{1 - |\Gamma_1\Gamma_L|}. \qquad (2)$$

For $\Gamma_{\min}$ to equal zero, $|\Gamma_1|$ must equal $|\Gamma_L|$. In practice, the range of $|\Gamma_L|$ that can be obtained is measured and then the bead is designed to give a reflection coefficient of the desired value. If a small reflection coefficient is of prime interest, the bead is made to have a reflection just slightly larger than the smallest obtainable value of $|\Gamma_L|$. If a wide range of reflection coefficient is desired, the bead is made to have a reflection coefficient just slightly smaller than the largest obtainable value of $|\Gamma_L|$.

The reflection coefficient of a single bead can be computed from (3).[4]

$$\Gamma_1 = \frac{-j(\epsilon - 1)\tan\dfrac{2\pi\sqrt{\epsilon}\,l}{\lambda}}{2\sqrt{\epsilon} + j(\epsilon - 1)\tan\dfrac{2\pi\sqrt{\epsilon}\,l}{\lambda}}, \qquad (3)$$

where Γ_1 is the reflection coefficient of the bead, l is the length of the bead, ϵ is the dielectric constant of the bead and λ is the wavelength in free space. If the value of

$$\tan\frac{2\pi\sqrt{\epsilon}\,l}{\lambda}$$

is small, which is usually the case, the magnitude of the reflection coefficient can be closely approximated by (4).

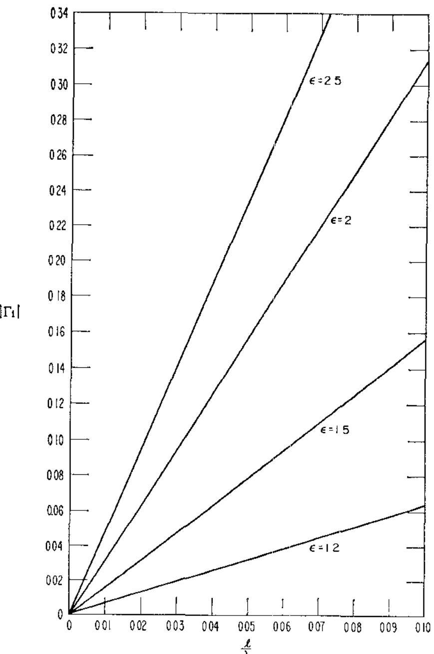

Fig. 2—Graph of (3).

$$\Gamma_1 \cong \frac{\pi l}{\lambda}(\epsilon - 1). \qquad (4)$$

Eq. (4) is graphed in Fig. 2 for values of ϵ from 1.2 to 2.5.

RESULTS

Figs. 3 and 4 show instruments that have been made using the above principles. The instrument in Fig. 3 was designed for a $\frac{3}{4}$-inch transmission line and the one in Fig. 4 was designed for a $\frac{9}{16}$-inch line. In each case, stable reflection coefficients of less than 0.005 were readily attained. The range of reflection coefficient of the terminating element of the $\frac{9}{16}$-inch instrument was from 0.02 to 0.15 at a frequency of 4 Gc. The bead was designed to give a reflection coefficient of approximately 0.05. This was done by making the outer diameter of the bead considerably less than the inner diameter of the outer conductor to give the bead an equivalent dielectric constant[5] of approxi-

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[1] W. J. Anson, "A guide to the use of the modified reflectometer technique of VSWR measurement," *J. Research Natl. Bur. Standards*, vol. 65C, pp. 217–223; October–December, 1961.

[2] R. C. Ellenwood and W. E. Ryan, "A UHF and MW matching termination," PROC. IRE, vol. 41, pp. 104–107; January, 1953.

[3] T. Moreno, "Microwave Transmission Design Data," Dover Publications, Inc., New York, N. Y., p. 31; 1958.

[4] *Ibid.*, see p. 84.

[5] J. W. E. Griesmann, "Handbook of Design and Performance of Cable Connectors for Microwave Use," Report No. R-520-56; PIB-450 for Bureau of Ships Contract No. NObsr-52078 Index No. NE-110718; 1956.

mately 1.5. If the lowest value of $|\Gamma_L|$ is sufficiently small, the reflection from the thin dielectric tube may be equal to, or larger than, this value and no bead is required to give a matched condition. The reflection from the thin dielectric tube can be calculated by first calculating the equivalent dielectric constant of the medium in the transmission line where the tube is located and then by computing $|\Gamma_1|$ from the equation

$$|\Gamma_1| = \frac{\sqrt{\epsilon}-1}{\sqrt{\epsilon}+1}. \qquad (5)$$

In Fig. 5, the quantity $|\Gamma_1|$ is plotted as a function of dielectric constant, over a realistic range of values. The use of a bead that only partially fills the coaxial line (or no bead at all) is desireable because it increases the stability of the instrument at very low values of reflection coefficient.

One significant result that has been obtained in using the very low-reflection sliding terminations is the capability to tune the reflectometer down to the point where the nonuniformities of the precision coaxial line become the limiting factor in the tuning operation, and hence the limiting factor in obtainable accuracy of a reflection coefficient measurement. In this light, the sliding termination should be very useful in evaluating the uniformity of coaxial lines as well as for improving the present obtainable accuracy of coaxial reflectometer measurements.

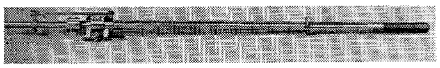

Fig. 3—¾-inch adjustable sliding termination.

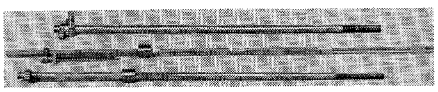

Fig. 4—$\frac{9}{16}$-inch adjustable sliding termination.

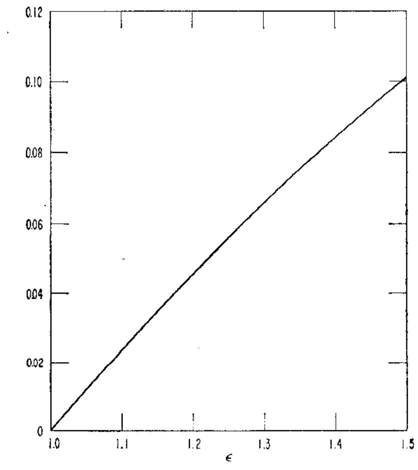

Fig. 5—Graph of (5).

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Approximate Method of Determining the Cutoff Frequencies of Waveguides of Arbitrary Cross Section

Theory

Electromagnetic propagation in a long, prismatic waveguide obeys the scalar Helmholtz equation

$$-\nabla^2\psi = K^2\psi$$

where

ψ = a potential function
K = frequency parameter
$-\nabla^2$ = positive definite plane Laplacian operator.

If the boundary is a curve natural to any of the common coordinate systems for which Helmholtz equation separates, the solutions can be derived by classical methods and may be expressed in terms of known transcendental functions. For the waveguides having more complicated cross sections, however, the cutoff frequencies can only be approximately determined. There are, however, some technical advantages in using these more complicated cross sections. In circular waveguides because of the axisymmetrical field configurations, the waves do not have directional stability but tend to shift in phase intermittently, producing fading and other undesirable consequences. To minimize this effect one or more logitudinal short vanes are sometimes installed on the wall to "lock" the modes in prescribed directions. These vanes change the cutoff frequencies in the waveguide by an appreciable amount. This correspondence shows that it is advantageous to conformally transform the complicated cross section onto a simpler one, (*i.e.*, the unit circle) where the boundary conditions can be easily satisfied. Transformation functions for many common curves are available in standard references.[1,2] For other curves, approximate transformation function can be determined by the series method.[2] Once the transformation function is known, the problem is reduced to the solution of the transformed equation as follows:

$$-\nabla^2\psi = \left|\frac{dz}{d\xi}\right|^2 K^2\psi \qquad (1)$$

where $z=f(\zeta)$: the transformation function. Many methods are available to solve the above equation approximately. Among them, the collocation method is perhaps the simplest. If greater accuracy of the approximate frequency parameter is desired, one may use the iteration technique suggested by Temple.[3] The first iteration can be expressed in terms of upper and lower bounds as follows:

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[1] L. V. Kantorovich, and V. I. Krylov, "Approximate Methods of Higher Analysis," Interscience Publishers, Inc., New York, N. Y.; 1958.

[2] N. I. Muskhelishvili, "Some Basic Problems on the Mathematical Theory of Elasticity," P. Noordhoff, Ltd., Groningen, Netherlands; 1953.

[3] G. Temple, "The computation of characteristic numbers and characteristic functions," *Proc. London Math. Soc.*, vol. 29, pp. 257–280; 1928.

$$\frac{\int_A u\cdot\nabla^2 u\,dA}{\int_A u^2 dA} - \frac{\dfrac{\int_A v^2 dA}{\int_A u^2 dA} - \left[\dfrac{\int_A u\nabla^2 u\,dA}{\int_A u^2 dA}\right]}{(K_2)^2 - \dfrac{\int_A u\nabla^2 u\,dA}{\int_A u^2 dA}} \le (K_1)^2 \le \frac{\int_A u\nabla^2 u\,dA}{\int_A u^2 dA} \qquad (2)$$

where u and v are two functions which satisfy the boundary conditions and are related by

$$-\nabla^2 u = v. \qquad (3)$$

K_2 is the estimated frequency parameter of the first harmonic and K_1 is the parameter which determines the lowest frequency cutoff point. In this discussion, only the case of the TM waves ($\psi=0$ at the boundary) is considered. The case of the TE waves merely requires a straight forward modification of the method.

Applications

A. *Rectangular Waveguide*

This case is treated here in order to illustrate the method. The transformation function which maps a square region whose sides are 2a by 2a onto a unit circle is

$$z = A\cdot a\int_0^{\xi}(1+\xi^4)^{-1/2}d\xi;\ A=1.08 \text{ and } \xi = re^{i\theta}.$$

The solution of the transformed partial differential equation can be expressed by means of a complete set of eigenfunctions

$$\psi(r,\theta) = Re\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{nm}J_n(k_{nm})e^{in\theta}$$

where k_{nm} satisfies the boundary condition

$$J_n(k_{nm}r)\big|_{r=1} = 0.$$

As a first approximation we take

$$\psi(r,\theta) \sim W(\mathrm{r}) = \sum_{m=0} B_{0m}J_0(K_{0m}r).$$

Substitution of the above function into (1) gives the error distribution,

$$\epsilon(r,\theta) = \sum_{m=1}^{N} B_{0m}J_0(k_{0m}r)$$
$$\cdot\left[k_{0m}{}^2(1+2r^4\cos 4\theta + r^8)^{1/2} + (1.08)^2a^2K^2\right].$$

We observe that $\epsilon(r,\theta)$ varies periodically with θ. For simplicity we assume that the mean error does not differ much from $(r, \pi/8)$. We arbitrarily choose five points. The computed first two frequency parameters are

$$K_1 = \frac{2.236}{a} \qquad K_2 = \frac{5.178}{a}$$

which compare favorably with the exact values from the closed form solutions which are

$$K_1 = \frac{2.2214}{a} \qquad K_2 = \frac{4.9673}{a}.$$